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General formalism in collinear regime for pseudoscalar meson production in NN collisions

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Abstract. The spin structure of the matrix element for the pseudoscalar meson production processes in nucleon-nucleon collisions is established in the collinear kinematical regime in terms of 3 independent scalar amplitudes. This result is valid for any reaction mechanism and for any energy of the colliding and the produced particles. The complete experiment for the full reconstruction of all 3 complex amplitudes must contain two different classes of polarization experiments. The polarization transfer coefficients can be used to determine the moduli of all 3 amplitudes, whereas the spin correlation coefficients for $\mathbf{p}+\mathbf{p}$ collisions are sensitive to the relative phases of different amplitudes.

PACS. 13.88.+e Polarization in interactions and scattering – 14.20.Dh Protons and neutrons

1 Introduction

Collinear kinematics is attractive. Because in this regime, for example, the unitarity condition for the elastic scattering results in the optical theorem, through which the imaginary part of the forward elastic amplitude can be related to the total cross-section for any energy. The conservation of the total helicity [1], being the general property of the collinear regime, simplifies the spin structure of the matrix element for any hadronic process as well as the polarization phenomena essentially.

The analysis of the spin structure and the polarization effects is controlled by the presence of a single physical kinematical direction for the collinear processes. Therefore, it is impossible to use here the results of the general analysis of the polarization phenomena for binary reactions. Because the essential ingredient of the general analysis is the existence of a scattering plane (which is produced by the various three-momenta in the initial and final states) and a definite coordinate system. The absence of the scattering plane for the collinear regime results in an axial symmetry which must be taken into account in introducing an adequate polarization formalism. Therefore collinear formalism must be developed independently and differently from that of ref. [2], because a continuous extrapolation from the general case to the collinear kinematics cannot be done in the framework of the above-mentioned general formalism. The number of independent collinear amplitudes is smaller than the one for the general case, which is the typical indication of noncontinuity.

It is clear that the polarization effects must be simplified essentially in the case of the axial symmetry. For example, all the one-spin T-odd observables must be zero for the collinear regime [3]. Let us also note that at high energies, the absolute values of the cross-sections are maximal in the collinear regime.

In this work we consider some special processes of pseudoscalar meson production in pp collisions, $p+p\to P+B+p$, where B is a baryon with spin parity $J^P=\frac{1}{2}^+$, and P is a pseudoscalar meson with $J^P=0^-$. There are several reasons for considering such processes [4,5]. Firstly, the nonbinary processes $1+2\to 3+4+5$ (where 1, 2, 3, 4, 5 denote hadrons) are especially interesting because the spin structure of the corresponding matrix elements is more complicated than the one for the binary processes $1+2\to 3+4$. So the most appropriate way to introduce the necessary generalization is to consider the simplest kinematical conditions, which evidently are the collinear ones.

Furthermore, it is the simplest processes of meson production in nucleon-nucleon collisons:

$$\begin{aligned} p+p &\to \pi^0+p+p\,,\\ &\to \eta(\eta')+p+p\,,\\ &\to K^+ +\Lambda(\Sigma)+p\,,\\ &\to \bar{D}+\Lambda_c+p\,. \end{aligned} \tag{1}$$

Note that the theoretical study of all these processes is very timely and appropriate due to the presence of many interesting physical questions, which can be solved with the help of these processes. Therefore these processes are at the center of experimental activity in different proton facilities (which operate at different energies), meson factories, SATURNE [6–9], DISTO [10], COSY [11–13], CELSIUS [14–16]. But general analysis of the spin structure of

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the matrix elements of these processes is not performed up to now in a model-independent way.

Therefore in this paper we analyze the spin structure of processes $p+p \to B+P+p$ in collinear regime, where there is a nontrivial, but simple enough spin structure, and this structure can be established in a model-independent form (independently of the details of the reaction mechanism). The possibility to find an adequate parametrization of collinear matrix element stimulates a discussion of another typical polarization problem, namely the problem of the "complete experiment". It will be the first attempt to consider this important problem for these nonbinary pr processes.

2 Parametrization of spin structure

The P-invariance of the strong interactions and the conservation of total helicity, valid for the collinear regime, implies that any process $p+p \to P+B+p$ is characterized by a set of 3 independent-helicity transitons:

$$\begin{aligned} p+p &\to B+P+p\,, \\ ++ &\to +0+\,, \\ +- &\to +0-\,, \\ &\to -0+\,, \end{aligned} \tag{2}$$

where $\pm$ denote the helicities of the baryons in the initial and final states being $\pm\frac{1}{2}$.

The simplest way to parametrize the spin structure of the matrix element in the most general form is to use the formalism of 2-component spinors in the CMS. Different equivalent parametrizations of spin structure for the collinear matrix can be introduced. We choose as starting point for our analysis of polarization phenomena the following construction for the general collinear matrix element:

$$\begin{aligned} \mathcal{M} = &\, g_1(\chi_2^+\chi_1)(\chi_4^+\sigma\cdot\mathbf{k}\chi_3) \\ &+g_2(\chi_2^+\sigma\cdot\mathbf{k}\chi_1)(\chi_4^+\chi_3) \\ &+ig_3(\chi_2^+\sigma_a\chi_1)[\chi_4^+(\sigma\times\mathbf{k})_a\chi_3]\,, \end{aligned} \tag{3}$$

where χ_1 and χ_2 are the 2-component spinors of colliding protons, χ_3 is the 2-component spinor of produced baryon B, χ_4 is the 2-component spinor of final proton, $\mathbf{k}$ is the unit vector along the direction of the proton beam, and g_1, g_2, g_3 are 3 independent collinear amplitudes. Note, that generally these amplitudes are complex functions of 3 possible independent kinematical variables which exist for $1+2 \to 3+4+5$ processes in the collinear regime. One can use as independent variables the following three energies: initial beam energy E, and the energies of the produced baryon B and the pseudoscalar meson P (note that these 3 variables fix all possible orientations of 3 final three-momenta, relative to each other).

Equation (3) corresponds to the so-called t-channel representation of the collinear spin structure, whose special form (namely, the "organization" of the order of 2-component spinors for all 4 baryons which are present in

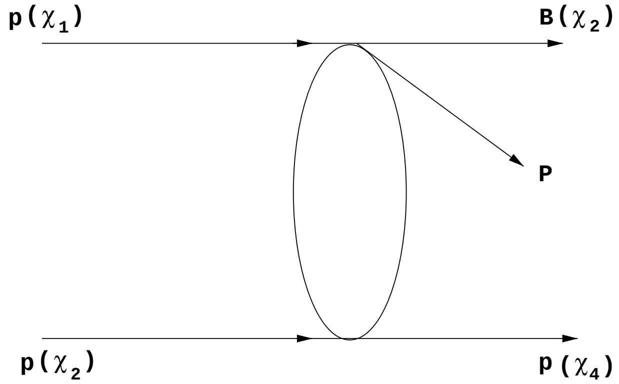

Fig. 1. Graphical representations of t-channel parametrizations of the spin structure of the matrix element for the processes $p+p \to B+P+p$.

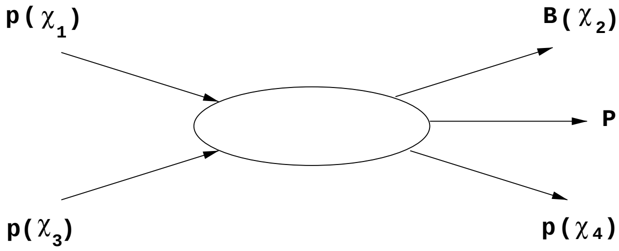

Fig. 2. Graphical representations of s-channel parametrizations of the spin structure of the matrix element for the processes $p+p \to B+P+p$.

the considered process, see fig. 1, is most suitable for the analysis of properties of different possible t-channel exchanges, which are considered usually as the most natural mechanism for the considered processes at high energy (the so-called peripheral collisions).

But sometimes another (equivalent) representation of the amplitude can also be useful, namely the so-called s-channel representation (fig. 2), with another arrangement of the order of baryonic spinors. This representation can be written as follows:

$$\begin{aligned} \mathcal{M} = &\, g_{1s}(\chi_2^+\sigma_y\tilde{\chi}_4^+)(\tilde{\chi}_3\sigma_y\sigma\cdot\mathbf{k}\chi_1) \\ &+g_{2s}(\chi_2^+\sigma\cdot\mathbf{k}\sigma_y\tilde{\chi}_4^+)(\tilde{\chi}_3\sigma_y\chi_1) \\ &+ig_{3s}(\chi_2^+\sigma_a\sigma_y\tilde{\chi}_4^+)[\tilde{\chi}_3\sigma_y(\sigma\times\tilde{k})_a\chi_1]\,, \end{aligned} \tag{4}$$

where g_{1s}, g_{2s}, g_{3s} are the corresponding scalar collinear amplitudes.

Using the standard Fierz transformations, the following relations between these two sets of collinear amplitudes, g_i and g_{is}, can be established:

$$\begin{aligned} g_1 &= \frac{1}{2}(g_{1s}+g_{2s}+g_{3s}), \\ g_2 &= \frac{1}{2}(g_{1s}+g_{2s}-g_{3s}), \\ g_3 &= \frac{1}{2}(2g_{1s}+2g_{2s}+g_{3s})\,. \end{aligned} \tag{5}$$

Note that the form presented in eq. (4) is especially useful for the analysis of pp collisions, from the point of view of possible s-channel mechanisms (dibaryon-like, or 6-quarks intermediate states). Such a picture of central, *i.e.* essentially nonperipheral mechanisms for pp collisions, is very effective for the description of different processes such as

$$\begin{aligned} p+p &\to \Delta+N, \\ &\to \Delta+\Delta, \\ &\to p+p+\pi^0, \\ &\to p+p+\eta, \end{aligned} \tag{6}$$

at intermediate energy region, $\tau_p \leq 2$ GeV.

It was found that [17,18] a limited number of special quantum states for pp-channel, namely $j^p = 2^+$ and 1^-, plays the most important role for such a s-channel mechanism. Therefore, the s-channel representation of the collinear matrix element, eq. (4), can be considered as the most suitable one for description of such two-baryon excitations. One can see that, the amplitude g_{1s} describes the triplet-singlet transition from initial pp system to final $p+B$ system, the amplitude g_{2s} describes a singlet-triplet transition, and the amplitude g_{3s} the triplet-triplet transition (with total-spin projection, equal to ± 1).

3 Polarization phenomena

So, to find the 3 moduli of the complex amplitudes $g_i, i = 1,2,3$, as well as the 2 relative phases, 5 different experiments most be performed for any reaction $p+p \to B+P+p$ in collinear conditions. In addition to the differential cross-section with unpolarized baryons in initial and final states, it is necessary to measure at least 4 different polarization observables. In the collinear regime, all possible one-spin polarization observables must be equal to zero identically. This is dictated by the kinematics and the symmetry properties and thus must be valid for any mechanism. Therefore, the simplest polarization observables which are nonzero in the collinear regime must be two-spin correlations of baryon polarizations. The first natural step of the complete experiment for $p+p \to B+P+p$ is the determination of the moduli of all 3 collinear amplitudes g_i, eq. (3). The direct experimental approach is to measure the coefficients of polarization transfer from initial to final baryon. For example, the dependence of polarization of produced baryon (3-vector of polarization $\mathbf{P}_2$, corresponding to spinor χ_2) on the polarization $\mathbf{P}_1$ of proton beam can be parametrized in the following general form (which is valid only for the collinear regime):

$$\mathbf{P}_2 = p_1\mathbf{P}_1 + p_2\mathbf{k}(\mathbf{k}\cdot\mathbf{P}_1), \tag{7}$$

where p_1 and p_2 are two real parameters, characterizing the standard coefficients of polarization transfer:

$$K_x^{x'} = K_y^{y'} = p_1\,, \qquad K_z^{z'} = p_1 + p_2\,, \tag{8}$$

if z-axis is chosen along the unit vector $\mathbf{k}$.

After some manipulations, the following expressions can be found for these coefficients (in terms of collinear amplitudes g_i):

$$\begin{aligned} p_1\left(\frac{\mathrm{d}\sigma}{\mathrm{d}\omega}\right)_0 &= |g_1|^2 - |g_2|^2\,, \\ p_2\left(\frac{\mathrm{d}\sigma}{\mathrm{d}\omega}\right)_0 &= 2(|g_2|^2 - |g_3|^2)\,, \end{aligned} \tag{9}$$

where $\mathrm{d}\omega$ is the definite element of the phase space for the 3-particle production (in the collinear regime).

We use here a special normalization of the collinear amplitudes, so the differential cross-section can be written in the following form:

$$\left(\frac{\mathrm{d}\sigma}{\mathrm{d}\omega}\right)_0 = |g_1|^2 + |g_2|^2 + 2|g_3|^2\,. \tag{10}$$

So, through the measurements of the following 3 observables, $(\mathrm{d}\sigma/\mathrm{d}\omega)_0, p_1$ and p_2, we can determine the moduli of all possible collinear amplitudes uniquely:

$$\begin{aligned} 4|g_1|^2 &= (1+3p_1+p_2)\left(\frac{\mathrm{d}\sigma}{\mathrm{d}\omega}\right)_0\,, \\ 4|g_2|^2 &= (1-p_1+p_2)\left(\frac{\mathrm{d}\sigma}{\mathrm{d}\omega}\right)_0\,, \\ 4|g_3|^2 &= (1-p_1-p_2)\left(\frac{\mathrm{d}\sigma}{\mathrm{d}\omega}\right)_0\,. \end{aligned} \tag{11}$$

Note that the polarization transfer between another pair of baryon, for instance between the target proton (with spinor χ_3, polarization vector $\mathbf{P}_3$) and the scattered proton (spinor χ_4, polarization vector $\mathbf{P}_4$), can be parametrized as

$$\mathbf{P}_4 = p_3\mathbf{P}_3 + p_4\mathbf{k}(\mathbf{k}\cdot\mathbf{P}_3). \tag{12}$$

This does not contain new physical infumation however, because the corresponding coefficients, *i.e.* the real parameters p_3 and p_4, are determined also by the moduli of amplitudes g_i:

$$\begin{aligned} p_3\left(\frac{\mathrm{d}\sigma}{\mathrm{d}\omega}\right)_0 &= |g_2|^2 - |g_1|^2\,, \\ p_4\left(\frac{\mathrm{d}\sigma}{\mathrm{d}\omega}\right)_0 &= 2(|g_1|^2 - |g_3|^2)\,. \end{aligned} \tag{13}$$

The following relations between two sets of parameters, p_1 and p_2, and p_3 and p_4, can easily be established:

$$p_3 = -p_2\,, \qquad p_4 = 2p_1 + p_2\,. \tag{14}$$

Note that the reactions $p+p \to K + \Lambda(\Sigma) + N$ are the most suitable ones measuring the transfer coefficients from the proton beam to the produced hyperon, because $\Lambda(\Sigma)$ are self-analyzing particles, and such an experiment has been carried out by DISTO collaboration [19–21].

To determine the relative phases of 3 complex amplitudes g_i, it is necessary to measure the spin correlation coefficients. The dependence of the differential cross-section on the polarizations $\mathbf{P}_1$ and $\mathbf{P}_3$ of colliding protons, which is valid at the level of P-invariance for the collinear kinematics, can be described as

$$\frac{d\sigma}{d\omega}(\mathbf{P}_1,\mathbf{P}_3)=\left(\frac{d\sigma}{d\omega}\right)_0(1+A_1\mathbf{P}_1\cdot\mathbf{P}_3+A_2\mathbf{k}\cdot\mathbf{P}_1\mathbf{k}\cdot\mathbf{P}_3)\,, \quad (15)$$

with the following expressions for the real coefficients A_1 and A_2, in terms of the t-channel collinear amplitudes g_i:

$$A_1 = 2\mathrm{Re}[(g_1-g_2)g_3^*]\,,$$
$$A_1+A_2 = 2\mathrm{Re}(g_1g_2^*)+|g_3|^2\,. \quad (16)$$

The sensitivity of these coefficients to the relative phases is evident from eq. (16). The relation between the coefficients A_i and the standard spin correlation coefficients C_{ab} are given as

$$C_{xx}=C_{yy}=A_1\,, \qquad C_{zz}=A_1+A_2\,. \quad (17)$$

But these observables are not sensitive to the signs of relative phases, $\delta_1-\delta_2$ and $\delta_1-\delta_3$, because the phase dependence of any T-even polarization observable has $\cos(\delta_1-\delta_2)$ and $\cos(\delta_1-\delta_3)$ form. Therefore, the observables with $\sin(\delta_1-\delta_2)$ and $\sin(\delta_1-\delta_3)$ dependence must be measured to have more or less a unique answer. This means that also the T-odd polarization observables must be measured.

But in the collinear regime the simplest T-odd polarization observables for the process, $p+p\to B+P+p$ must include the triple polarization correlations. As an example, the dependence of the polarization $\mathbf{P}_2$ of the produced baryon on the polarizations $\mathbf{P}_1$ and $\mathbf{P}_3$ (for colliding protons) can be parametrized in the following general form:

$$\mathbf{P}_2 = t_1\mathbf{P}_1\times\mathbf{P}_3+t_2\mathbf{k}(\mathbf{k}\cdot\mathbf{P}_1\times\mathbf{P}_3)+t_3(\mathbf{k}\times\mathbf{P}_1\mathbf{k}\cdot\mathbf{P}_3$$
$$+\mathbf{k}\times\mathbf{P}_3\mathbf{k}\cdot\mathbf{P}_1)\,, \quad (18)$$

where t_i are real independent coefficients which are determined by the imaginary parts of the definite combinations of the collinear scalar amplitudes

$$t_1\frac{d\sigma}{d\omega}=2\mathrm{Im}(g_1g_3^*)\,,$$
$$t_2\frac{d\sigma}{d\omega}=2\mathrm{Im}[g_1(g_2-g_3)^*]\,,$$
$$t_3\frac{d\sigma}{d\omega}=2\mathrm{Im}(g_2g_3^*)\,, \quad (19)$$

in which T-odd nature is manifest.

Note that all other possible triple correlations of the baryonic polarizations can be expressed as definite linear combinations of parameters t_i because, generally, there are only 3 independent T-odd products of the collinear amplitudes for the considered processes.

Before concluding our discussion of possible polarization phenomena in collinear NN interactions, we would like to note that the coefficients A_1 and A_2 can be expressed also in terms of the s-channel collinear amplitudes $g_{is}, i=1,2,3$:

$$A_1\left(\frac{d\sigma}{d\omega}\right)_0 = -|g_{1s}|^2+|g_{2s}|^2\,,$$
$$A_2\left(\frac{d\sigma}{d\omega}\right)_0 = -2|g_{2s}|^2+2|g_{3s}|^2\,, \quad (20)$$

Note that there are no interference contributions here. But such interference terms, $\mathrm{Re}g_{is}g_{js}^*$, $i,j=1,2,3$, will be present in coefficients p_1 and p_2(p_3 and p_4), which define the spin transfer coefficients.

4 Conclusions

We would like to summarize here the main results of our work:

- We established the spin structure of collinear matrix element for a wide class of interesting pseudoscalar meson production processes (covering a wide spectrum from π meson to η_c or D) in proton-proton collisions, $p+p\to B+P+p$, in terms of 3 complex amplitudes.
- We performed the general analysis of polarization phenonena in the collinear regime, and established the content of the complete experiment for the full reconstruction of the spin structure.

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